Quark induced excitations of the instanton liquid

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Abstract

The selfconsistent approach to the quark interactions in the instanton liquid is developed within the tadpole approximation calculating the basic functional integral. The effective Lagrangian obtained includes colourless scalar field interacting with quarks. The origin of this dynamical field as an interaction carrier in soft momentum region is discussed.

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I. Introduction

Nowadays, there are no doubts the model of QCD vacuum as the instanton liquid (IL) is the most practical instrument on the chiral scale of QCD. It provides, as the lattice calculations recently confirmed, not only the theoretical background for describing spontaneous chiral symmetry breaking (SCSB) but is mostly powerful in the phenomenology of the QCD vacuum and in the physics of light quarks while considered to propagate by zero modes arising from instantons. The origin of gluon and chiral condensates turns out in this picture easily understandable and both are quantitatively calculated getting very realistic values defined by Λ_{QCD} and parameters of instanton and anti-instanton ensemble, for example, $-i \langle \psi^\dagger \psi \rangle \sim -(250 \ MeV)^3$. Moreover, the scale for dynamical quark masses, $M \sim 350 \ MeV$, naturally appears and pion decay constant, $F_\pi \sim 100 \ MeV$, is then transparently calculated.

Another significant advantage of this approach is that the initial formulation starts basically from the first principles and subsequent approximations being well grounded and reliably controlled are plugged in [1], [2]. It becomes clear especially in recent years when the impressive progress has been reached in understanding the instanton physics on the lattice [3]. Further we are summarizing several things we have learned thinking of the IL theory and trying to answer the challenging questions.

Let us start on that stage of the IL approach when its generating functional has already been factorized as

$$\mathcal{Z} \ = \ \mathcal{Z}_g \ \cdot \ \mathcal{Z}_\psi \ ,$$

where eventually the factor $\mathcal{Z}_g$ provides nontrivial gluon condensate while the fermion part $\mathcal{Z}_\psi$ is responsible to describe the chiral condensate in instanton medium and its excitations. It is usually supposed the functional integral of $\mathcal{Z}_g$ is saturated by the superposition of the pseudo-particle (PP) fields which are the Euclidean solutions of the Yang-Mills equations called the (anti-)instantons

$$A_\mu(x) = \sum_{i=1}^{N} A_\mu(x; \gamma_i) \ . \tag{1}$$

Here $A_\mu(x; \gamma_i)$ denotes the field of a single (anti-)instanton in singular gauge with $4N_c$ (for the $SU(N_c)$ group) coordinates $\gamma = (\rho, z, U)$ of size ρ centred at the coordinate z and colour orientation defined by the matrix U. The nontrivial bloc of corresponding $N_c \times N_c$ matrices of PP is a part of potential

$$A_\mu(x; \gamma) = \frac{\bar{\eta}_{a\mu\nu}}{g} \frac{y_\nu}{y^2} \frac{\rho^2}{y^2 + \rho^2} \ U^\dagger \tau_a \ U, \quad y = x - z \ , \quad a = 1, 2, 3 \ , \tag{2}$$

where τ_a are the Pauli matrices, η is the 't Hooft symbol [4], g is the coupling constant and for anti-instanton $\bar{\eta} \to \eta$. For the sake of simplicity we do not introduce the distinct symbols for instanton (N_+) and anti-instanton (N_-) and consider topologically neutral IL with $N_+ = N_- = N/2$. Formulating the variational principle the practical estimate of $\mathcal{Z}_g$ was found [2]

$$\mathcal{Z}_g \ \simeq \ e^{-\langle S \rangle}$$

with the action of IL defined by the following additive functional [1]

$$\langle S \rangle = \int dz \int d\rho \ n(\rho) \ s(\rho) \ . \tag{3}$$

The integration should be performed over the IL volume V along with averaging the action per one instanton

$$s(\rho) = \beta(\rho) + 5\ln(\Lambda\rho) - \ln \widetilde{\beta}^{2N_c} + \beta\xi^2\rho^2 \int d\rho_1 \ n(\rho_1)\rho_1^2, \tag{4}$$

weighted with instanton size distribution function

$$n(\rho) = C \ e^{-s(\rho)} = C \ \rho^{-5} \widetilde{\beta}^{2N_c} e^{-\beta(\rho) - \nu\rho^2/\overline{\rho^2}} \ , \tag{5}$$

$$\nu = \frac{b - 4}{2}, \ b = \frac{11 \ N_c - 2 \ N_f}{3} \ ,$$

where $\overline{\rho^2} = \int d\rho \ \rho^2 \ n(\rho)/n = \left(\frac{\nu}{\beta\xi^2 n}\right)^{1/2}$, $n = \int d\rho \ n(\rho) = N/V$ and N_f is the number of flavours. The constant C is defined by the variational maximum principle in the selfconsistent way and $\beta(\rho) = \frac{8\pi^2}{g^2} = -\ln C_{N_c} - b\ln(\Lambda\rho)$ $(\Lambda = \Lambda_{\overline{MS}} = 0.92\Lambda_{P.V.})$ with constant C_{N_c} depending on the renormalization scheme, in particular, here $C_{N_c} \approx \frac{4.66 \ \exp(-1.68N_c)}{\pi^2(N_c - 1)!(N_c - 2)!}$. The parameters $\beta = \beta(\bar{\rho})$ and $\widetilde{\beta} = \beta + \ln C_{N_c}$ are fixed at the characteristic scale $\bar{\rho}$ (an average instanton size). The constant $\xi^2 = \frac{27}{4} \frac{N_c}{N_c^2 - 1} \pi^2$ characterizes, in a sense, the PP interaction and Eqs. (3),(4) and (5) describe the equilibrium state of IL. The minor modification of variational maximum principle (see Appendix) leads to the explicit form of the mean instanton size $\bar{\rho}\Lambda = \exp\left\{-\frac{2N_c}{2\nu - 1}\right\}$ and, therefore, to the direct definition of the IL parameters unlike the conventional variational principle [2] which allows one to extract those parameters solving numerically the transcendental equation only.

The quark fields are considered to be *influenced* by the certain stochastic ensemble of PPs, Eq. (1), while calculating the quark determinant

$$\mathcal{Z}_\psi \ \simeq \ \int D\psi^\dagger D\psi \ \langle\langle \ e^{S(\psi,\psi^\dagger,A)} \ \rangle\rangle_A \ .$$

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[1] In fact, the additive property results from the supposed homogeneity of vacuum wave function in metric space. Eq. (3) looks like a formula of classical physics although it describes the ground state of quantum instanton ensemble. Intuitively clear, this definition will be still valid even when the wave function is nonhomogeneous with the nonuniformity scale essentially exceeding average instanton size or, precisely speaking, being larger (or of the order) than average size of characteristic saturating field configuration. Then each instanton liquid element of such a distinctive size will provide a partial contribution depending on the current state of IL, see next Section.

Besides, dealing with the dilute IL (small characteristic packing fraction parameter $n \bar{\rho}^4$) one neglects the correlations between PPs and utilizes the approximation of $N_c \to \infty$ where the planar diagrams only survive. In addition the fermion field action is approached by the zero modes which are the solutions of the Dirac equation $i(\hat{D}(A_\pm) + m) \, \Phi_\pm = 0$ in the field of (anti-)instanton $A_\pm$, i.e.

$$[\Phi_\pm(x)]_{ic} = \frac{\rho}{\sqrt{2}\pi|x|(x^2 + \rho^2)^{3/2}} \left[\hat{x} \, \frac{1 \pm \gamma_5}{2}\right]_{ij} \varepsilon_{jd} \, U_{dc} \tag{6}$$

with the colour c, d and the Lorentz i, j indices and the antisymmetric tensor ε. In particular, at $N_f = 1$ the quark determinant reads [2]

$$\mathcal{Z}_\psi \simeq \int D\psi^\dagger D\psi \, \exp\left\{\int dx \, \psi^\dagger(x) \, i\hat{\partial}\psi(x)\right\} \, \left(\frac{Y^+}{VR}\right)^{N_+} \left(\frac{Y^-}{VR}\right)^{N_-} , \tag{7}$$

$$Y^\pm = i \int dz \, dU \, d\rho \, n(\rho)/n \int dxdy \, \psi^\dagger(x) \, i\hat{\partial}_x \Phi_\pm(x - z) \, \Phi_\pm^\dagger(y - z) \, i\hat{\partial}_y \, \psi(y) ,$$

where the factor R makes the result dimensionless and is also fixed by the saddle point calculation. Much more accurate result for the Green function of quark in the PP ensemble [6] allows one to calculate the generating functional even beyond the chiral limit and the simple zero mode approximation turns out amazingly fruitful again to develop (quantitatively) the low energy phenomenology of light quarks [7].

Thus, the IL approach at the present stage of its development looks very indicative, well theoretically grounded and reasonably adjusted phenomenologically. The proper form of generating functional obtained and its reasonable parameter dependence provide with enough predictive power and justify, hence, the approximations made. Generally, it leads to conclude the quark feedback upon the instanton background is pretty limp and could be perturbatively incorporated as a small variation of instanton liquid parameters δn and $\delta \rho$ around their equilibrium values of n and $\bar{\rho}$ being in full analogy with the description of chiral condensate excitations. Indeed, the result of nontrivial calculation of the functional integral (treating the zero quark modes in the fermion determinant substantially from physical view point) takes down to 'encoding' the IL state with just those two parameters. Moreover, the IL density appears in the approach via the packing fraction parameter $n\bar{\rho}^4$ only (clear from dimensional analysis) what means one independent parameter existing in practice. It is just the instanton size. The analysis of the *quark and IL interaction* is addressed in this paper developing our idea [8] of phononlike excitations of IL resulting from the adiabatic changes of the instanton size. Hence, we describe further the *quark feedback* dealing with these deformable (anti-)instanton configurations which are the field configurations Eq. (2) characterized by the size ρ depending on x and z, i.e. $\rho \to \rho(x, z)$.

The paper is organized as follows: in Section II we discuss the modification of quark determinant when the functional integral is saturated by the deformable modes at one flavour. Then in Section III we develop the proper calculation (tadpole approximation) which is based on the saddle point method. Section IV is devoted to the generalization for the multiflavour picture. The paper includes also Appendix where the fault finding reader gets a chance to control the explicit formulae for the IL parameters and to improve our calculations if is able.

II. Dynamics of phononlike excitations

Apparently, the essence of what we discuss here could be illuminated in the following way. Saddle point method calculation of the functional integral implies the treatment of the action extremals which are the solutions of classical field equations. For the case in hands the action $S[A, \psi^\dagger, \psi]$ is constructed including the gluon fields A_μ, (anti-)quarks $\psi^\dagger, \psi$ and extremals which are given by the solutions of the consistent system of the Yang-Mills and Dirac equations. As a trial configuration

in the IL theory the superposition of (anti-)instantons which is the approximate solution of the Yang-Mills equations (with no backreaction of the quark fields) and a background field for the Dirac equation is simultaneously considered. We believe it is reasonable to make use of the deformable (crumpled) (anti-)instantons $A_\pm(x; \gamma(x))$ as the saturating configurations. They just admit of varying the parameters $\gamma(x)$ of the Yang-Mills sector of the initial consistent system in order to describe the influence of quark fields in the appropriate variables for the quark determinant.

Taking the action in the form $S[A_\pm(x, \gamma(x)), \psi^\dagger, \psi]$ we would receive the corresponding variational equation for the deformation field $\gamma(x)$ which approaches most optimally (as to the action extremum) PP at nonzero quark fields. In the field theory, for example, the monopole scattering [9], the Abrikosov vortices scattering [10] are treated in a similar way. Actually, the unstationary picture seems more adequate as a specific role of instanton-anti-instanton annihilation channel prompts and an investigation of two-body colour problem teaches [11].

In practice, for IL we avoid the difficulties which come with solving the variational equations if we consider the long wavelength excitations only (with the wave length λ much larger than the characteristic instanton size $\bar\rho$). Indeed, it can be done because we are searching the kinetic energy of the deformation fields [2] (the one particle contributions) and consider the pair interaction which develops the contact interaction form being calculated in the adiabatic regime [8].

Let us remember here that deriving Eq. (3) we should average over the instanton positions in a metric space. Clearly, the characteristic size of the domain L which has to be taken into account should exceed the mean instanton size $\bar\rho$. But at the same time it should not be too large because the far ranged elements of IL are not 'causally' related. The ensemble wave function is expected to be homogeneous (every PP contributes to the functional integral being weighted with a factor proportional to $\sim 1/V$, $V = L^4$) on this scale. The characteristic configuration which saturates the functional integral is taken as the superposition Eq. (1) with N number of PP in the volume V. It is easy to understand that because of an additivity of the functional, Eq.(3) describes properly even non-equilibrium states of IL when the distribution function $n(\rho)$ does not coincide with the vacuum one and, moreover, allows us to generalize this action for the non-homogeneous liquid when the size of the homogeneity obeys the obvious requirement $\lambda \geq L > \bar\rho$. In particular, the ensemble of deformable PPs as saturating the functional integral obeys these constraints if the corrections coming with the deformation fields $g_{\mu\nu}$ are small and the long wavelength excitations (on the scale of the instanton size $\bar\rho$, i.e. $\left|\dfrac{\partial\rho(x, z)}{\partial x}\right| \ll O(1)$) of initial instanton fields $G_{\mu\nu}$ are mainly taken into consideration. Such a condition of smoothness or the adiabatic change of instanton size dictates, in practice, another essential simplification to define the field in the center of instanton $\dfrac{\partial\rho(x, z)}{\partial x} \sim \left.\dfrac{\partial\rho(x, z)}{\partial x}\right|_{x=z}$ as a characteristic deformation field in further calculations.

In principle, the correction to the instanton field produced by the quark presence might be calculated in its most general form, if the gluon Green function in the instanton medium is known, as

$$a_\mu^a(x, z) = \int d\xi \, D_{\mu\nu}^{ab}(x - z, \xi - z) \, J_\nu^b(\xi - z_\psi) , \qquad (8)$$

where J_ν^b is the current of external quark source, z_ψ belongs to the region of long wavelength disturbance and, at last, $D_{\mu\nu}^{ab}$ is the Green function of PP in the instanton medium. In fact, this Green function (even when considered in the field of one instanton) is not well defined [5] but, seems, for the case in hands we could develop the selfconsistent way to calculate the regularized Green function. The nonsingular propagator behaviour in the soft momentum region is defined by the mass gap of phononlike excitations. Fortunately, the exact form of the Green function occurs to be not of great importance here (we are planning to return to the problem of regularized Green function calculation in the forthcoming publication). In the coordinate space it is peaked around the average PP size

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[2]Then we are allowed to take the slowly changing deformation field beyond the integral while calculating the action of deformed instanton.

being in nonperturbative regime and, hence, integral Eq. (8) is estimated to be

$$a_\mu^a(x, z) \simeq \bar{D}_{\mu\nu}^{ab}(x - z) \ \bar{J}_\nu^b(z - z_\psi) \ . \tag{9}$$

On the other hand, the exact instanton definition in the singular gauge,

$$A_\mu^a(x, z) = -\frac{\bar{\eta}_{a\mu\nu}}{g} \frac{\partial}{\partial x_\nu} \ln\left(1 + \frac{\rho^2}{y^2}\right) \ ,$$

leads to the following correction to the instanton potential

$$a_\mu^a(x) = H_{\mu\nu}^a(x, z) \ \frac{\partial\rho}{\partial x_\nu} \ , \tag{10}$$

where $H_{\mu\nu}^a(x, z) = -\frac{\bar{\eta}_{a\mu\nu}}{g} \frac{2\rho(x, z)}{y^2 + \rho^2(x, z)}$. Confining within the precision accepted here we could take $\rho(x, z) \simeq \bar{\rho}$, i.e. $H_{\mu\nu}^a(x, z) = H_{\mu\nu}^a(x - z)$.

If we compare now both definitions of the correction we are capable to get immediately for ρ_ν the following equation

$$H_{\mu\nu}^a(x - z) \ \frac{\partial\rho(x, z)}{\partial x_\nu} = \bar{D}_{\mu\nu}^{ab}(x - z) \ \bar{J}_\nu^b(z - z_\psi) \ .$$

In fact, the current $\bar{J}$ might be taken constant in the long wavelength approximation. Then neglecting the current gradients one is allowed to change the derivatives to obtain the following estimate of the deformation velocity $\left|\frac{\partial\rho(x, z)}{\partial x}\right| \sim \left|\frac{\partial\rho(z)}{\partial z}\right|$ which looks to be well justified since there is no other fields in the problem at all (in the adiabatic approximation). The contribution of deformed (anti-)instantons to the functional integral (when the corrections coming from the PP deformation fields are absorbed) may be estimated as [8]

$$\langle S \rangle \simeq \int dz \int d\rho \ n(\rho) \left\{ \frac{\kappa}{2} \left(\frac{\partial\rho}{\partial z}\right)^2 + s(\rho) \right\} \ ,$$

where κ is the kinetic coefficient being derived within the quasiclassical approach. Our estimate of it gives the value of a few single instanton actions $\kappa \sim c \beta$ with the coefficient $c \sim 1.5 \div 6$ depending quantitatively on the ansatz supposed for the saturating configurations. Although this estimate is not much meaningful because there is no the vital κ dependence eventually (becomes shortly clear) and the kinetic term could be introduced phenomenologically. Thus, this coefficient should be fixed on a characteristic scale, for example $\kappa \sim \kappa(\bar{\rho})$, if we not are planning to be beyond the precision peculiar to the approach. Actually, it means adding the small contribution of kinetic energy type to the action per one instanton only. Such a term results from the scalar field of deformations and affects negligibly the pre-exponential factors of the functional integral. In one's turn the pre-exponential factors do the negligible influence on the kinetic term as well. The deformation fields induced by the dilations and rotations in the isotopic space result in the singular and trivial kinetic coefficients, respectively [8]. Then to evaluate the mass scale related to those modes one needs the heuristic ideas rooted, apparently, beyond the conventional IL and SCSB.

If we strive for being within the approximation developed we should retain the small terms of the second order in deviation from the point of action minimum $\left.\frac{ds(\rho)}{d\rho}\right|_{\rho=\rho_c} = 0$ only expecting the approximate validity of

$$s(\rho) \simeq s(\bar{\rho}) + \frac{s^{(2)}(\bar{\rho})}{2} \ \varphi^2 \ , \tag{11}$$

where $s^{(2)}(\bar{\rho}) \simeq \left.\dfrac{d^2 s(\rho)}{d\rho^2}\right|_{\rho_c} = \dfrac{4\nu}{\bar{\rho}^2}$ and the scalar field $\varphi = \delta\rho = \rho - \rho_c \simeq \rho - \bar{\rho}$ is the field of deviations from the equilibrium value of $\rho_c = \bar{\rho}\left(1 - \dfrac{1}{2\nu}\right)^{1/2} \simeq \bar{\rho}$. Consequently, the deformation field is described by the following Lagrangian density

$$\mathcal{L} = \frac{n\kappa}{2}\left\{\left(\frac{\partial\varphi}{\partial z}\right)^2 + M^2\varphi^2\right\}$$

with the mass gap of the phononlike excitations

$$M^2 = \frac{s^{(2)}(\bar{\rho})}{\kappa} = \frac{4\nu}{\kappa\overline{\rho^2}}$$

which is estimated for IL with $N_c = 3$, for example, in the quenched approximation to be

$$M \approx 1.21 \; \Lambda$$

if $c = 4$, $\bar{\rho}\,\Lambda \approx 0.37$, $\beta \approx 17.5$, $n\,\Lambda^{-4} \approx 0.44$ (for the details see the tables of Appendix).

Changing the variables we obtain the gluon part of the generating functional as

$$\mathcal{Z}'_g \sim \int D\varphi \left|\frac{\delta A}{\delta\varphi\cdots}\right| \exp\left\{-\frac{n\kappa}{2}\int dz\left[\left(\frac{\partial\varphi}{\partial z}\right)^2 + M^2\varphi^2\right]\right\} ,$$

with the Jacobian $\left|\dfrac{\delta A}{\delta\varphi\cdots}\right|$ corresponding those new variables introduced. Let us notice that the latter looks pretty formal because of incomplete set of the transformations shown. However, in the adiabatic approximation, as was mentioned above, the preexponential Jacobian contribution to the action [3] being a c-number should be omitted.

Analyzing the modifications which arise now in the quark determinant $\mathcal{Z}_\psi$ we take into account the variation of fermion zero modes resulting from the instanton size perturbed

$$\Phi_\pm(x - z, \rho) \simeq \Phi_\pm(x - z, \rho_c) + \Phi_\pm^{(1)}(x - z, \rho_c)\,\delta\rho(x, z) ,$$

where $\Phi_\pm^{(1)}(u, \rho_c) = \left.\dfrac{\partial\Phi_\pm(u, \rho)}{\partial\rho}\right|_{\rho=\rho_c}$ and because of the adiabaticity it is valid $\delta\rho(x, z) \simeq \delta\rho(z, z) = \varphi(z)$. The additional contributions of scalar field generate the corresponding corrections in the factors of the kernels $Y^\pm$ of Eq. (7) which are treated in the linear approximation in φ and taking approximately $\rho_c = \bar{\rho}$, i.e.

$$i\hat{\partial}_x\Phi_\pm(x - z, \rho)\,\Phi_\pm^\dagger(y - z, \rho)\,i\hat{\partial}_y \simeq \Gamma_\pm(x, y, z, \bar{\rho}) + \Gamma_\pm^{(1)}(x, y, z, \bar{\rho})\,\varphi(z) , \qquad (12)$$

here we introduced the notations

$$\Gamma_\pm(x, y, z, \bar{\rho}) = i\hat{\partial}_x\Phi_\pm(x - z, \bar{\rho})\Phi_\pm^\dagger(y - z, \bar{\rho})(-i\hat{\partial}_y) ,$$

$$\Gamma_\pm^{(1)}(x, y, z, \bar{\rho}) = i\hat{\partial}_x\Phi_\pm^{(1)}(x - z, \bar{\rho})\Phi_\pm^\dagger(y - z, \bar{\rho})\,(-i\hat{\partial}_y) + i\hat{\partial}_x\Phi_\pm(x - z, \bar{\rho})\,\Phi_\pm^{\dagger(1)}(y - z, \bar{\rho})\,(-i\hat{\partial}_y)$$

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[3]Generally, the deformation field ρ_ν and integration variable a_μ^a (10) are related via the rotation matrix: $\tilde{a}_\mu^a = \Omega_{ab}\Phi_{\mu\nu}^b(x - z)\rho_\nu$ and in the long-length wave approximation Φ might be constant $\Phi_{\mu\nu}^b(0)$ $(x \sim z)$. With the rotation matrix spanning the colour field $a_\mu = \Omega^{-1}\tilde{a}_\mu$ on the fixed axis we can conclude that the vectors a_μ^z and ρ_ν are, in fact, in one to one correspondence (of course, being within one loop approximation and up to this unessential colour rotation). Thus, the Jacobian contribution turns out to be an unessential constant.

with $-i\hat{\partial}_y$ left acting operator (the gradients of scalar field φ are negligible according to the adiabaticity assumption again). It is a simple matter to verify that the right hand side of Eq. (12), being integrated over $dzdU$, generates the following kernel (in the momentum space)

$$\frac{1}{N_c}\left[(2\pi)^4\,\delta(k-l)\,\gamma_0(k,k) + \gamma_1(k,l)\,\varphi(k-l)\right] \tag{13}$$

with $\gamma_0(k,k) = G^2(k)$, $G(k) = 2\pi\bar{\rho}F(k\bar{\rho}/2)$, $\gamma_1(k,l) = G(k)G'(l) + G'(k)G(l)$,
$G'(k) = \left.\dfrac{dG(k)}{d\rho}\right|_{\rho=\bar{\rho}}$, $F(x) = 2x\,[I_0(x)K_1(x) - I_1(x)K_0(x)] - 2\,I_1(x)K_1(x)$, where I_i, K_i ($i=0,1$) are the modified Bessel functions.

The functional integral of Eq. (7) including the phononlike component may be exponentiated in the momentum space [4] with the auxiliary integration over the λ-parameter (see, for example [2])

$$\mathcal{Z}_\psi \simeq \int \frac{d\lambda}{2\pi}\,\exp\left\{N\ln\left(\frac{N}{i\lambda VR}\right) - N\right\} \times$$
$$\times \int D\psi^\dagger D\psi\,\exp\left\{\int \frac{dkdl}{(2\pi)^8}\,\psi^\dagger(k)\left[(2\pi)^4\delta(k-l)\left(-\hat{k} + \frac{i\lambda}{N_c}\gamma_0(k,k)\right) + \frac{i\lambda}{N_c}\gamma_1(k,l)\,\varphi(k-l)\right]\psi(l)\right\}$$

(we dropped out the factor normalizing to the free Lagrangian everywhere). It is pertinent to mention here the Diakonov-Petrov result comes to the play precisely if the scalar field is switched off.

In order to avoid a lot of the needless coefficients in the further formulae we introduce the dimensionless variables (momenta, masses and vertices)

$$\frac{k\bar{\rho}}{2} \to k\,, \quad \frac{M\bar{\rho}}{2} \to M\,, \quad \gamma_0 \to \bar{\rho}^2\gamma_0\,, \quad \frac{1}{(n\bar{\rho}^4\kappa)^{1/2}}\,\gamma_1 \to \bar{\rho}\gamma_1\,, \tag{14}$$

the fields in turn

$$\varphi(k) \to (n\kappa)^{-1/2}\bar{\rho}^3\varphi(k)\,, \quad \psi(k) \to \bar{\rho}^{5/2}\psi(k)\,, \tag{15}$$

and eventually for λ we are using $\mu = \dfrac{\lambda\bar{\rho}^3}{2N_c}$. Then the generating functional takes the following form

$$\mathcal{Z} \simeq \int d\mu\,\mathcal{Z}_g''\int D\psi^\dagger D\psi\,D\varphi\,\exp\left\{n\bar{\rho}^4\left(\ln\frac{n\bar{\rho}^4}{\mu} - 1\right) - \int \frac{dk}{\pi^4}\,\frac{1}{2}\,\varphi(-k)\,4\,[k^2 + M^2]\,\varphi(k)\right\} \times$$
$$\times \exp\left\{\int \frac{dkdl}{\pi^8}\,\psi^\dagger(k)\,2\,\left[\pi^4\delta(k-l)(-\hat{k} + i\mu\gamma_0(k,k)) + i\mu\,\gamma_1(k,l)\,\varphi(k-l)\right]\psi(l)\right\}\,, \tag{16}$$

where $\mathcal{Z}_g''$ is a part of gluon component of the generating functional which survives after expanding the action per one instanton Eq. (11). The functional obtained describes the IL state influenced by the quarks when all the terms containing the scalar field are collected (see also Appendix). As mentioned above we believe this influence analogous to the back impact of phononlike deformations on the quark determinant does not considerably change the numerical results of the IL and SCSB

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[4] In the metric space we have the nonlocal Lagrangian of the phononlike deformations $\varphi(z)$ interacting with the quark fields $\psi^\dagger$, ψ, i.e.

$$\mathcal{L} = \int dx\,\psi^\dagger(x)i\hat{\partial}_x\psi(x) - \int dz\,\frac{n\kappa}{2}\left\{\left(\frac{\partial\varphi}{\partial z}\right)^2 + M^2\varphi^2(z)\right\} +$$
$$+ \frac{i\lambda_\pm}{N_c}\int dxdydz\,dU\,\psi^\dagger(x)\{\Gamma_\pm(x,y,z,\bar{\rho}) + \Gamma_\pm^{(1)}(x,y,z,\bar{\rho})\,\varphi(z)\}\psi(y)\,.$$

The physical meaning of the basic phenomenon behind this Lagrangian seems pretty transparent. The propagation of quark fields through the instanton medium is accompanied by the IL disturbance (the analogy with well known polaron problem embarrasses us strongly in this point).

theory. Noninteracting part of phononlike excitation Lagrangian characterizes the IL reaction on the external long-length wave perturbation and, apparently, is its general feature independent of perturbating field nature.

The presence of quark condensate makes hint for the appropriate scheme of approximate calculation of the generating functional

$$\psi^\dagger \psi \; \varphi = \langle \psi^\dagger \psi \rangle \; \varphi + \{ \psi^\dagger \psi - \langle \psi^\dagger \psi \rangle \} \; \varphi \; .$$

III. Tadpole approximation

The formal integration over the scalar field leads us to the four fermion interaction and the functional integral can not be calculated exactly. However, due to smallness of scalar field corrections we may find the effective Lagrangian substituting the condensate value in lieu of one of the pairs of quark lines (see Fig. 1a.)

$$\psi^\dagger(k)\psi(l) \rightarrow \langle \psi^\dagger(k)\psi(l) \rangle = -\pi^4 \delta(k - l) \; Tr \; S(k)$$

where $S(k)$ is the quark Green function. In such an approach the diagram with four fermion lines in the lowest order of the perturbation theory in μ is reduced to the two-legs diagram with one tadpole contribution (there are two such contributions because of two possible ways of pairing)

$$2 \; (i\mu)^2 \int \frac{dkdl \; dk'dl'}{\pi^{16}} \; \gamma_1(k,l) \; \gamma_1(k',l') \;\; \psi^\dagger(k)\psi(l) \;\; \psi^\dagger(k')\psi(l') \;\; \langle \varphi(k-l) \; \varphi(k'-l') \rangle \simeq$$

$$\simeq 4 \; \mu^2 \int \frac{dk}{\pi^4} \; \gamma_1(k,k) \; \psi^\dagger(k)\psi(k) \; \int \frac{dl}{\pi^4} \; \gamma_1(l,l) \; Tr \; S(l) \; D(0) \; ,$$

where the natural pairing definition was introduced

$$\langle \varphi(k) \; \varphi(l) \rangle = \pi^4 \delta(k + l) \; D(k), \;\; D(k) = \frac{1}{4 \; (k^2 + M^2)} \; .$$

It is obvious the factors surrounding $\psi^\dagger(k)\psi(k)$ have a meaning of an additional contribution to the dynamical mass

$$m(k) = \mu \; \gamma_1(k,k) \; (-2i\mu) \int \frac{dl}{\pi^4} \; \gamma_1(l,l) \; Tr \; S(l) \; D(0) \; , \tag{17}$$

(the initial mass term contains the factor 2 when the dimensionless variables are utilized, i.e. takes a form $2im$.)

The contribution of the graph with all the quark lines paired (see Fig. 1b)

$$-2 \; \mu^2 \left[\int \frac{dk}{\pi^4} \; \gamma_1(k,k) \; Tr \; S(k) \right]^2 \pi^4 \delta(0) D(0) = -\frac{\mu^2}{2} \frac{V}{\bar\rho^4} \frac{\kappa}{\nu} \left[\int \frac{dk}{\pi^4} \; \gamma_1(k,k) \; Tr \; S(k) \right]^2 ,$$

together with contribution of the graph (Fig. 1c)

$$2 \; \mu^2 \int \frac{dkdl}{\pi^8} \;\; Tr\gamma_1(k,l) \; \gamma_1(l,k) \; S(k) \; S(l) \; D(k - l) \; ,$$

should be taken into account at the same order of the μ expansion while calculating the saddle point equation. Here we used the natural regularization of the δ-function $\delta(0) = \frac{1}{\pi^4} \frac{V}{\bar\rho^4}$ in the dimensionless units. Then the quark determinant after integrating over the scalar field reads

$$\mathcal{Z} \sim \int d\mu \int D\psi^\dagger D\psi \;\; \exp \left\{ n\bar\rho^4 \left(\ln \frac{n\bar\rho^4}{\mu} - 1 \right) + \frac{2N_c^2}{n\bar\rho^4 \nu} \frac{V}{\bar\rho^4} \; \mu^4 \; c^2(\mu) - \right.$$

$$-2N_c\mu^2\frac{V}{\bar{\rho}^4}\int\frac{dkdl}{\pi^8}\ \gamma_1^2(k,l)\frac{(kl)-\Gamma(k)\Gamma(l)}{(k^2+\Gamma^2(k))(l^2+\Gamma^2(l))}D(k-l)+\int\frac{dk}{\pi^4}\ \psi^\dagger(k)\ 2\ [-\hat{k}+i\Gamma(k)]\ \psi(k)\Big\}=$$

$$=\int d\mu\ \exp\Big\{n\bar{\rho}^4\left(\ln\frac{n\bar{\rho}^4}{\mu}-1\right)+\frac{2N_c^2}{n\bar{\rho}^4\nu}\frac{V}{\bar{\rho}^4}\ \mu^4\ c^2(\mu)-$$

$$-\ 2N_c\mu^2\frac{V}{\bar{\rho}^4}\int\frac{dkdl}{\pi^8}\ \gamma_1^2(k,l)\frac{(kl)-\Gamma(k)\Gamma(l)}{(k^2+\Gamma^2(k))(l^2+\Gamma^2(l))}D(k-l)+\frac{V}{\bar{\rho}^4}\ \int\frac{dk}{\pi^4}\ \text{Tr}\ \ln[-\hat{k}+i\Gamma(k)]\Big\}\ ,$$

(18)

where the vertex function is defined as

$$\Gamma(k)=\mu\ \gamma_0(k,k)+m(k)\ ,$$

and we introduced the function $c(\mu)$ convenient for the practical calculations

$$c(\mu)=-\frac{i(n\bar{\rho}^4\kappa)^{1/2}}{2\mu\ N_c}\ \int\frac{dk}{\pi^4}\ \gamma_1(k,k)\ \text{Tr}\ S(k)\ .$$

As seen from Eq. (18) the Green function of the quark field is selfconsistently defined by the following equation

$$2\ [-\hat{k}+i\Gamma(k)]\ S(k)=-1\ .$$

Searching the solution in the form

$$S(k)=\ A(k)\ \hat{k}+i\ B(k)\ ,$$

we get

$$A(k)=\frac{1}{2}\ \frac{1}{k^2+\Gamma^2(k)}\ ,\qquad B(k)=\frac{1}{2}\ \frac{\Gamma(k)}{k^2+\Gamma^2(k)}\ .$$

Using Eq. (17) and the definitions of $\Gamma(k)$ and $B(k)$ we have the complete integral equation

$$\Gamma(k)=\mu\ \gamma_0(k,k)+N_c\frac{\kappa}{\nu}\ \mu^2\ \gamma_1(k,k)\ \int\frac{dl}{\pi^4}\ \gamma_1(l,l)\ \frac{\Gamma(l)}{l^2+\Gamma^2(l)}\ ,$$

which drives to have the convenient representation of the solution

$$\Gamma(k)=\mu\ \gamma_0(k,k)+\frac{N_c}{(n\bar{\rho}^4\kappa)^{1/2}}\frac{\kappa}{\nu}\ \mu^3\ c(\mu)\ \gamma_1(k,k)\ .$$

What concerns the function $c(\mu)$ it is not a great deal to obtain

$$c(\mu)=\frac{(n\bar{\rho}^4\kappa)^{1/2}}{\mu}\ \int\frac{dk}{\pi^4}\ \gamma_1(k,k)\ \frac{\Gamma(k)}{k^2+\Gamma^2(k)}\ ,$$

and, therefore, the complete integral equation for the function $c(\mu)$ which is shown in Fig. 2 for $N_f=1$. Let us underline the N_f-dependence of the $c(\mu)$ function in the interval of μ determined by saddle point value is unessential. Then we easily obtain for the additional contribution to the dynamical quark mass

$$m(k)=\frac{N_c}{(n\bar{\rho}^4\kappa)^{1/2}}\frac{\kappa}{\nu}\ \mu^3\ c(\mu)\ \gamma_1(k,k)\ ,$$

(19)

and see the cancellation of the kinetic coefficient κ in m if we remember the definition (14). Thus, it means the precise value of the coefficient is unessential as declared.

We have the following equation for the saddle point of the functional of Eq. (18)

$$\frac{n\bar{\rho}^4}{\mu}-2N_c\int\frac{dk}{\pi^4}\ \frac{[\Gamma^2(k)]'_\mu}{k^2+\Gamma^2(k)}+2N_c\int\frac{dkdl}{\pi^8}\ \left\{\frac{\mu^2\gamma_1^2(k,l)[(kl)-\Gamma(k)\Gamma(l)]}{(k^2+\Gamma^2(k))(l^2+\Gamma^2(l))}\right\}'_\mu D(k-l)-$$

$$-\frac{2N_c^2}{n\bar{\rho}^4\nu}\ [\mu^4\ c^2(\mu)]'_\mu-(n\bar{\rho}^4)'_\mu\ \ln\frac{n\bar{\rho}^4}{\mu}=0\ ,$$

(20)

where the prime is attributed to the differentiation in μ. Being interested in receiving a closed equation we need to know the derivative $c'(\mu)$ too. Then the definition of $c(\mu)$ above allows us to have

$$(1 - \mu^2 A(\mu)) \ c'(\mu) = 2\mu \ A(\mu) \ c(\mu) + B(\mu) \ ,$$

where

$$A(\mu) = \alpha(\mu) \ \frac{N_c \kappa}{\nu} \int \frac{dk}{\pi^4} \frac{\gamma_1^2(k)}{k^2 + \Gamma^2(k)} \frac{k^2 - \Gamma^2(k)}{k^2 + \Gamma^2(k)} \ ,$$

$$B = -\frac{2(n\bar\rho^4 \kappa)^{1/2}}{\mu^2} \int \frac{dk}{\pi^4} \frac{\gamma_1(k) \ \Gamma^3(k)}{(k^2 + \Gamma^2(k))^2} \ ,$$

and

$$\alpha(\mu) = 1 - \mu^2 \frac{N_c}{\beta\xi^2} \frac{\Gamma(\nu + 1/2)}{\nu^{1/2}\Gamma(\nu)} \ \frac{c(\mu)}{n\bar\rho^4 \left(n\bar\rho^4 - \frac{\nu}{2\beta\xi^2} \right)} \ .$$

Thereby the saddle point equation absorbs the effect of the IL parameter modification ($n(\mu)$) produced by equilibrium instanton size shift $\rho_c \sim \bar\rho$ which comes in the leading order from simple tadpole graph

$$2 \ i\mu \int \frac{dkdl}{\pi^8} \ \gamma_1(k,l) \ (-\pi^4) \ \delta(k - l) \ Tr \ S(k) \ \varphi(k - l) = \Delta \cdot \ \varphi(0) \ ,$$

$$\Delta = -2i\mu \ \int \frac{dk}{\pi^4} \ \gamma_1(k,k) \ Tr \ S(k) = \frac{4N_c}{(n\bar\rho^4 \kappa)^{1/2}} \ \mu^2 c(\mu) \ , \tag{21}$$

(remind here $\varphi = \rho - \rho_c$, and $\varphi(0) = \int dz \ \varphi(z)$ is the scalar field in momentum representation).

In the Table 1 the numerical results (M.S.Z.) are shown for $N_f = 1$ comparing to those Diakonov and Petrov (D.P.)

Table 1.

D.P.			M.S.Z.		
μ	$M(0)$	$-i\langle\psi^\dagger\psi\rangle$	μ	$M(0)$	$-i\langle\psi^\dagger\psi\rangle$
$5.27 \cdot 10^{-3}$	359	$-(333)^3$	$4.81 \cdot 10^{-3}$	362	$-(322)^3$

The parameters indicated in the Table 1 are
the dynamical quark mass

$$M(0) = 2\Gamma(0) \ \left(\frac{1}{\bar\rho}\right) \quad [MeV]$$

and the quark condensate

$$-i\langle\psi^\dagger\psi\rangle = i \ Tr \ S(x)|_{x=0} = -2N_c \ \int \frac{dk}{\pi^4} \frac{\Gamma(k)}{k^2 + \Gamma^2(k)} \ \left(\frac{1}{\bar\rho}\right)^3 \quad [MeV]^3 \ .$$

Through this paper the value of renormalization constant is fixed by $\Lambda = 280 \ MeV$. Then the IL parameters are slightly different from their conventional values $\bar\rho \sim (600 \ MeV)^{-1}$, $\bar R \sim (200 \ MeV)^{-1}$ (see the corresponding tables in Appendix). However, with the minor Λ variation the parameters could be optimally fitted. As expected the change of quark condensate is insignificant, the order of several MeV, what hints the existence of new soft energy scale established by the disturbance which accompanies the quark propagation through the instanton medium.

IV. Multiflavour approach

In order to match the approach developed to phenomenological estimates we need the generalization for $N_f > 1$. Then the quark determinant becomes [2], [12]

$$\mathcal{Z}_\psi \simeq \int D\psi^\dagger D\psi \; \exp\left\{\int dx \; \sum_{f=1}^{N_f} \psi_f^\dagger(x) \; i\hat{\partial}\psi_f(x)\right\} \; \left(\frac{Y^+}{VR^{N_f}}\right)^{N_+} \left(\frac{Y^-}{VR^{N_f}}\right)^{N_-} ,$$

$$Y^\pm = i^{N_f} \int dz \; dU \; d\rho \; n(\rho)/n \prod_{f=1}^{N_f} \int dx_f dy_f \; \psi_f^\dagger(x_f) \; i\hat{\partial}_{x_f}\Phi_\pm(x_f - z) \; \Phi_\pm^\dagger(y_f - z) \; i\hat{\partial}_{y_f} \; \psi_f(y_f) .$$

With phononlike component included every pair of the zero modes $\sim \Phi \; \Phi^\dagger$ acquires the additional term similar to Eq. (12). The appropriate transformation driving the factors $Y^\pm$ to their determinant forms [2] is still valid here since the correction term differs from the basic one with the scalar field φ. The complete integration over dz leads (in the adiabatic approximation $\varphi(x,z) \to \varphi(z)$) to the transparent Lagrangian form with the momentum conservation of all interacting particles. Besides, we keep the main terms of $Y^\pm$ expanding in φ. The quark zero modes generate the factor similar to Eq. (13) with $\frac{1}{N_c}$ being changed by the factor $\left(\frac{1}{N_c}\right)^{N_f}$ and then in the leading N_c order we have

$$Y^\pm = \left(\frac{1}{N_c}\right)^{N_f} \int dz \; \det_{N_f}\left(i \; J^\pm(z)\right) ,$$

$$J_{fg}^\pm(z) = \int \frac{dkdl}{(2\pi)^8} \left[e^{i(k-l)z}\gamma_0(k,l) + \int \frac{dp}{(2\pi)^4} \; e^{i(k-l+p)z}\gamma_1(k,l) \; \varphi(p)\right] \; \psi_f^\dagger(k) \; \frac{1\pm\gamma_5}{2} \; \psi_g(l) .$$

While providing the functional with the Gaussian form we perform the integration over the auxiliary parameter λ together with the bosonization resulting in the integration over the auxiliary matrix $N_f \times N_f$ meson fields [12]

$$\exp\left[\lambda \; \det\left(\frac{i \; J}{N_c}\right)\right] \simeq \int d\mathcal{M} \; \exp\left\{i \; Tr[\mathcal{M}J] - (N_f - 1) \left(\frac{\det[\mathcal{M}N_c]}{\lambda}\right)^{\frac{1}{N_f-1}}\right\} .$$

As a result the generating functional may be written as

$$\mathcal{Z} = \int \frac{d\lambda}{2\pi} \; \mathcal{Z}_g'' \; \exp(-N \ln \lambda) \; \int D\varphi \; \exp\left\{-\int \frac{dk}{(2\pi)^4} \; \frac{n\kappa}{2} \; \varphi(-k) \; [k^2 + M^2] \; \varphi(k)\right\} \cdot$$

$$\cdot \int D\mathcal{M}_{L,R} \; \exp \int dz \left\{-(N_f - 1)\left[\left(\frac{\det[\mathcal{M}_L N_c]}{\lambda}\right)^{\frac{1}{N_f-1}} + \left(\frac{\det[\mathcal{M}_R N_c]}{\lambda}\right)^{\frac{1}{N_f-1}}\right]\right\} \cdot \qquad (22)$$

$$\cdot \int D\psi^\dagger D\psi \; \exp\left\{\int \frac{dk}{(2\pi)^4} \sum_f \psi_f^\dagger(k)(-\hat{k})\psi_f(k) + i \; \int dz \left(Tr[\mathcal{M}_L J^+] + Tr[\mathcal{M}_R J^-]\right)\right\} .$$

Now scalar field interacts with the quarks of the different flavours, nevertheless, the dominant contribution is expected from the tadpole graphs where any pair of the quark fields is taken in the condensate approximation as done at $N_f = 1$

$$\psi_f^\dagger(k)\psi_g(l) \to \langle\psi_f^\dagger(k)\psi_g(l)\rangle = -\pi^4\delta_{fg} \; \delta(k - l) \; Tr \; S(k) .$$

As for the condensate itself we obtain it as the nontrivial solution of saddle point equation. For example, it is

$$(\mathcal{M}_{L,R})_{fg} = \mathcal{M} \; \delta_{fg}$$

for the diagonal meson fields. The dimensionless convenient variables (in addition to Eqs. (14), (15)) are the following

$$\frac{\mathcal{M}}{2}\,\bar{\rho}^3 \to \mu\ , \qquad \left(\frac{\lambda\bar{\rho}^4}{(2N_c\bar{\rho})^{N_f}}\right)^{\frac{1}{N_f-1}} \to g\ .$$

Then the effective action ($\mathcal{Z} = \int dg d\mu\ \exp\{-V_{eff}\}$) in new designations has the form

$$V_{\text{eff}} = N(N_f - 1)\ln g - \frac{V}{\bar{\rho}^4}(N_f - 1)\frac{2\mu^{\frac{N_f}{N_f-1}}}{g} - \frac{V}{\bar{\rho}^4}\frac{2N_f^2 N_c^2}{n\bar{\rho}^4\,\nu}\mu^4 c^2(\mu) +$$
$$+2N_c N_f\mu^2\frac{V}{\bar{\rho}^4}\int\frac{dkdl}{\pi^8}\,\gamma_1^2(k,l)\frac{(kl)-\Gamma(k)\Gamma(l)}{(k^2+\Gamma^2(k))(l^2+\Gamma^2(l))}D(k-l) - 2N_f N_c\,\frac{V}{\bar{\rho}^4}\,\int\frac{dk}{\pi^4}\ln\{k^2+\Gamma^2(k)\}\ , \tag{23}$$

and the saddle point equation reads

$$\frac{n\bar{\rho}^4}{\mu} - 2N_c\int\frac{dk}{\pi^4}\,\frac{[\Gamma^2(k)]'_\mu}{k^2+\Gamma^2(k)} - \frac{2N_c^2 N_f}{n\bar{\rho}^4\nu}[\mu^4\ c^2(\mu)]'_\mu +$$
$$+2N_c\int\frac{dkdl}{\pi^8}\left\{\frac{\mu^2\gamma_1^2(k,l)[(kl)-\Gamma(k)\Gamma(l)]}{(k^2+\Gamma^2(k))(l^2+\Gamma^2(l))}\right\}'_\mu D(k-l) - (n\bar{\rho}^4)'_\mu\ln\left(\left(\frac{n\bar{\rho}^4}{2}\right)^{1/N_f}\frac{1}{\mu}\right) = 0\ . \tag{24}$$

The additional contribution to the dynamical quark mass in Eq. (19) gains the factor N_f because the scalar nature of the phononlike field requires to match the tadpole quark field condensates of all N_f flavours to each vertex

$$m(k) = \frac{N_f N_c}{(n\bar{\rho}^4\kappa)^{1/2}}\frac{\kappa}{\nu}\,\mu^3\ c(\mu)\ \gamma_1(k,k)\ .$$

Table 2 complements the Table 1 with the calculations at $N_f = 2$

Table 2.

D.P.					M.S.Z.				
μ	$M(0)$	$-i\langle\psi^\dagger\psi\rangle$	F_π	F'_π	μ	$M(0)$	$-i\langle\psi^\dagger\psi\rangle$	F_π	F'_π
$4.83\cdot10^{-3}$	386	$-(381)^3$	135	111	$3.26\cdot10^{-3}$	298	$-(335)^3$	108	90

where $F_\pi\ [MeV]$ is the pion decay constant and

$$F_\pi^2 = \frac{N_c N_f}{2}\ \int\frac{dk}{\pi^4}\,\frac{\Gamma^2(k) - \frac{k}{2}\Gamma'(k)\Gamma(k) + \frac{k^2}{4}(\Gamma'(k))^2}{(k^2+\Gamma^2(k))^2}\ \left(\frac{1}{\bar{\rho}}\right)^2\ ,$$

$F'_\pi\ [MeV]$ is its approximated form

$$F'^2_\pi = \frac{N_c N_f}{2}\ \int\frac{dk}{\pi^4}\frac{\Gamma^2(k)}{(k^2+\Gamma^2(k))^2}\ \left(\frac{1}{\bar{\rho}}\right)^2\ ,$$

here $\Gamma'(k) = \dfrac{d\Gamma(k)}{dk}$, and the condensate $-i\langle\psi^\dagger\psi\rangle$ is implied for the quarks of each flavour.

The light particle introduced and which imitates the scalar glueball properties does not affect significantly the SCSB parameters and correctly describes the soft pion excitations of quark condensate. Meanwhile, the experimental status of this light scalar glueball is very vague. We believe the phononlike excitations could manifest themselves being mixed, for instance, with the excitations of the quark condensate in the scalar channel.

VI. Conclusion

In this paper we have developed the consistent approach to describe the interaction of quarks with IL. Theoretically, it is based (and justified) on the particular choice of the configurations saturating the functional integral what is not merely a technical exercise. They are the deformable (crumpled) (anti-)instantons with the variable parameters $\gamma(x)$ and in the concrete treatment of this paper we play with the variation of the PP size $\rho(x, z)$. In a sense, such an ansatz is strongly motivated by the form of quark determinant which is solely dependent on the average instanton size in the SCSB theory. We have demonstrated that in the long-length wave approximation the variational problem of the deformation field optimization turns into the construction of effective Lagrangian for the scalar phononlike φ and quark fields with the Yukawa interaction. Physically, it allows us to analyze the backreaction of quarks on the instanton vacuum. We have pointed out this influence on the IL parameters as negligible. The modification of the SCSB parameters turns out pretty poor as well. In particular, the scale of quark condensate change amounts to a few MeV only. Nevertheless, switching on the phononlike excitations of IL leads to several qualitatively new and interesting effects. The propagation of the quark condensate disturbances over IL happens in this approach to be in close analogy with well-known polaron problem. We imply a necessity to take into account the medium feedback while elementary excitations propagating. Besides, it hints that fitting the parameters $\bar{\rho}$, n and renormalization constant Λ all together with the alteration of $s(\rho)$ profile function we might achieve suitable agreement not only in the order of magnitude. The difficulties which confronted us here illuminate the fundamental problem of gluon field penetration into the vacuum (the instanton vacuum in this particular case) as the most principle one. Indeed, it is a real challenge to answer the question about the strong interaction carrier in the soft momentum region. Perhaps, the light particle of scalar glueball properties which appears inherently in our approach and should manifest itself in the mixture with the excitation of quark condensate in scalar channel (σ-meson) is not bad candidate for that role. By the way, it could be experimentally observed [5] as a wide resonance.

Summarizing, we understand our calculation can not pretend to the precise quantitative agreement with experimental data and see many things to be done. We are planning shortly to consider the problem of instanton profile [13], to make more realistic description of the PP interaction and to push our ansatz beyond the long-length wave approximation analyzing more precisely 'instanton Jacobian' $\left|\frac{\delta A}{\delta \varphi}\right|$.

Appendix

The contribution of the quark determinant to the IL action is given by the tadpole diagram Eq. (21) which takes the following form when returned to the dimensional variables (see, Eq. (15))

$$\Delta \varphi = \Delta \, \frac{(n\kappa)^{1/2}}{\bar{\rho}^3} \, \varphi(0) = \Delta \, (n\bar{\rho}^4 \kappa)^{1/2} \int d\rho \frac{n(\rho)}{n} \, \int \frac{dz}{\bar{\rho}^4} \, \frac{\rho(z) - \rho_c}{\bar{\rho}} \ .$$

---

[5] We receive the effective Lagrangian of Yukawa type which admits the estimate of quark-anti-quark bound state while adapted to the non-relativistic approximation and the inequality $\frac{\mu\gamma_0(0)}{4\pi} \frac{\mu^2\gamma_1^2(0)}{M} \geq 2$ if valid signals its appearance. In the problem under consideration the left hand side of the inequality is $O(1)$.

Then the IL action, Eq. (3), acquires the additional term

$$\langle S \rangle = \int dz \, n \left\{ \langle s \rangle - \langle \Delta' \, \frac{\rho - \rho_c}{\bar{\rho}} \rangle \right\} \,,$$

where $\Delta' = \frac{4N_c}{n\bar{\rho}^4} \, \mu^2 c(\mu)$ and the mean action per one instanton is given by the following functional $\langle s_1 \rangle = \int d\rho \, s_1(\rho)n(\rho)/n$ with

$$s_1(\rho) = \beta(\rho) + 5\ln(\Lambda\rho) - \ln \tilde{\beta}^{2N_c} + \beta\xi^2\rho^2 n\overline{\rho^2} - \Delta' \, (\rho - \rho_c)/\bar{\rho} \,.$$

In order to evaluate the equilibrium parameters of IL we treat the maximum principle

$$\langle e^{-S} \rangle \geq \langle e^{-S_0} \rangle e^{-\langle S - S_0 \rangle}$$

adapting it to the simplest version (when the approximating functional is trivial $S_0 = 0$). In a sense, this choice of the approximating functional should be a little 'worse' than in Ref. [2]. Its only advantage comes from the possibility to get the explicit formulae for the IL parameters in lieu of solving the complicated transcendental equation. In equilibrium the instanton size distribution function $n(\rho)$ should be dependent on the IL action only, i.e. $n(\rho) = Ce^{-s(\rho)}$ where C is a certain constant. This argument corresponds to the maximum principle of Ref. [2]. Indeed, if one is going to approach the functional (3) as a local form $\langle s \rangle = \int d\rho \, s_1(\rho)n(\rho)/n$ where $s_1(\rho) = \beta(\rho) + 5\ln(\Lambda\rho) - \ln \tilde{\beta}^{2N_c} + \beta\xi^2\rho^2 n\overline{\rho^2}$, it makes the approach selfconsistent. The functional difference $\langle s \rangle - \langle s_1 \rangle = \int d\rho \, \{s(\rho) - s_1(\rho)\}e^{-s(\rho)}/n$ being varied over $s(\rho)$ leads then to the result $s(\rho) = s_1(\rho) + const$ keeping into the mind an arbitrary normalization. The maximum principle results in getting the mean action per one instanton as the IL parameters function, for instance, average instanton size $\bar{\rho}$. The corrections generated by the 'shifting' terms turn out to be small and we consider them in the linear approximation in the deviation Δ. The following schematic expansion exhibits how the major contribution appears

$$\langle s_1 \rangle = \frac{\langle (s + \delta) \, e^{-s-\delta} \rangle}{\langle e^{-s-\delta} \rangle} \simeq \frac{\langle s \, e^{-s} \rangle + \langle \delta \, e^{-s} \rangle}{\langle e^{-s} \rangle} + \frac{\langle s \, e^{-s} \rangle \langle \delta \, e^{-s} \rangle - \langle s \, \delta \, e^{-s} \rangle \langle e^{-s} \rangle}{\langle e^{-s} \rangle^2} \,, \qquad (25)$$

here $\delta(\Delta)$ stands for a certain small 'shifting' contribution and s is the action generated by the gluon component only. The last term in Eq. (25) is small comparing to the first one and we ignore it. Then it is clear that evaluating the mean action per one instanton is permissible to hold the gluon condensate contribution s only (without the 'shifting' term δ) in the exponential. Hence, we have for the mean action per one instanton $\langle s_1 \rangle = \int d\rho \, s_1(\rho)n_0(\rho)/n_0$, and $n_0(\rho)$ is the distribution function which does not include the 'shifting' term [6]. It is possible to obtain for the average squared instanton size and the IL density that [7]

$$r^2\overline{\rho^2} = \nu \left\{ 1 + \frac{\Delta'}{r\bar{\rho}} \frac{\Gamma(\nu + 1/2)}{2\nu\Gamma(\nu)} \right\} \simeq \nu \left\{ 1 + \Delta' \frac{\Gamma(\nu + 1/2)}{2\nu^{3/2}\Gamma(\nu)} \right\} \,, \qquad (26)$$

---

[6]The 'shifting' term changes the mass of phononlike excitation insignificantly. The equilibrium instanton size as dictated by the condition $\left. \frac{ds(\rho)}{d\rho} \right|_{\rho = \rho_c} = 0$ is equal to $\rho_c = (\alpha + \Delta' \, \beta) \, \bar{\rho}$, $\alpha = \left(1 - \frac{1}{2\nu} \right)^{1/2}$, $\beta = \frac{1}{4\nu} \left\{ 1 - \alpha \frac{\Gamma(\nu + 1/2)}{\nu^{1/2}\Gamma(\nu)} \right\}$ and the second derivative of action in the equilibrium point equals to $s''(\rho_c) = \frac{4\nu}{\rho^2} + \frac{2\nu}{\rho^2} \Delta' \left\{ \frac{\Gamma(\nu + 1/2)}{\nu^{3/2}\Gamma(\nu)} - \frac{1}{2\nu\alpha} \right\}$. Another source of corrections to the kinetic coefficient appears while one considers the instanton profile change $A \to A + a$ where the field of correction is $a \sim \left. \frac{\partial\rho(x, z)}{\partial x} \right|_{x=z}$. This mode could appear within the superposition ansatz Eq. (1) and leads to the modification of quark zero mode $(D(A + a)\psi = 0)$. Fortunately, both corrections to the kinetic term are numerically small.

[7]In order not to overload the formulae with the factors making the results dimensionless, which are proportional to the powers of Λ, we drop them out hoping it does not lead to the misunderstandings.

$$n = C \ C_{N_c} \tilde{\beta}^{2N_c} \frac{\Gamma(\nu)}{2r^{2\nu}} \ , \tag{27}$$

where the parameter r^2 equals to

$$r^2 = \beta \xi^2 n \overline{\rho^2} \ . \tag{28}$$

Expanding $\ln \rho = \ln \bar{\rho} + \frac{\rho - \bar{\rho}}{\bar{\rho}} + \frac{1}{2} \frac{(\rho - \bar{\rho})^2}{\bar{\rho}^2} + \cdots$ and using Eq. (26) we can show that

$$\frac{\int d\rho \ n_0(\rho) \ln \rho}{\int d\rho \ n_0(\rho)} = \ln \bar{\rho} + \Phi_1(\nu) \ , \quad \frac{\int d\rho \ n_0(\rho)\rho}{\int d\rho \ n_0(\rho)} = \bar{\rho} + \Phi_2(\nu) \ ,$$

where Φ_1, Φ_2 are the certain function of ν independent of $\bar{\rho}$. Besides, the average squared instanton size within the precision accepted obeys the equality $r^2 \overline{\rho^2} = \Phi(\nu)$, and $\Phi(\nu)$ is the function of ν only. Then the mean action per one instanton looks like

$$\langle s_1 \rangle = -2N_c \ln \tilde{\beta} + (2\nu - 1) \ \ln \bar{\rho} + F(\nu)$$

($F(\nu)$ is again the function of ν only and its explicit form is unessential here). Calculating its maximum in $\bar{\rho}$ we receive

$$\bar{\rho} = \exp \left\{ -\frac{2N_c}{2\nu - 1} \right\} \ , \quad \beta = \frac{2bN_c}{2\nu - 1} - \ln C_{N_c} \ .$$

Eqs. (26), (28) allows us to receive the equation for packing fraction parameter

$$(n\bar{\rho}^4)^2 - \frac{\nu}{\beta \xi^2} \ n\rho^4 = \frac{\Delta}{\beta \xi^2} \frac{\Gamma(\nu + 1/2)}{2\sqrt{\nu} \ \Gamma(\nu)} \ . \tag{29}$$

and for positive root we find

$$n = \nu \ \frac{e^{\frac{8N_c}{2\nu - 1}}}{\beta \xi^2} \left\{ 1 + \Delta' \frac{\Gamma(\nu + 1/2)}{2\nu^{3/2}\Gamma(\nu)} \right\} \ ,$$

and handling Eq. (27) we determine the constant C.

The IL parameters come about close to the parameter values of the Diakonov-Petrov approach [2] and are shown in the following Table

Table 3.

	D.P.				M.S.Z.		
N_f	$\bar{\rho}\Lambda$	n/Λ^4	β	N_f	$\bar{\rho}\Lambda$	n/Λ^4	β
0	0.37	0.44	17.48	0	0.37	0.48	17.48
1	0.30	0.81	18.86	1	0.33	0.70	18.11
2	0.24	1.59	20.12	2	0.28	1.13	18.91

Here N_f is the number of flavours ($N_f = 0$ corresponds to the quenched approximation) and $N_c = 3$. It is curious to notice the quark influence on the IL equilibrium state provokes the increase of the IL density.

In Table 4 we demonstrate the mass gap magnitude M and the wave length in the 'temporal' direction $\lambda_4 = M^{-1}$. To make it more indicative we show also the average distance between PPs from which it is clear, indeed, that the adiabatic approximation $\lambda \geq L \sim \bar{R} > \bar{\rho}$ is valid for the long-length wave excitations of the π-meson type. All the parameters are taken at $\kappa = 4\beta$ but the primed ones correspond to the kinetic term value of $\kappa = 6\beta$.

Table 4.

N_f	$M\Lambda^{-1}$	$\lambda\Lambda$	$M'\Lambda^{-1}$	$\lambda'\Lambda$	$\bar{R}\Lambda$
0	1.21	0.83	0.99	1.01	1.2
1	1.34	0.75	1.09	0.91	1.1
2	1.45	0.69	1.18	0.84	0.97

here $\bar{R} = n^{-1/4}$ is the distance between PPs.

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a)   b)

c)

——————— fermion fields ψ, $\psi^\dagger$
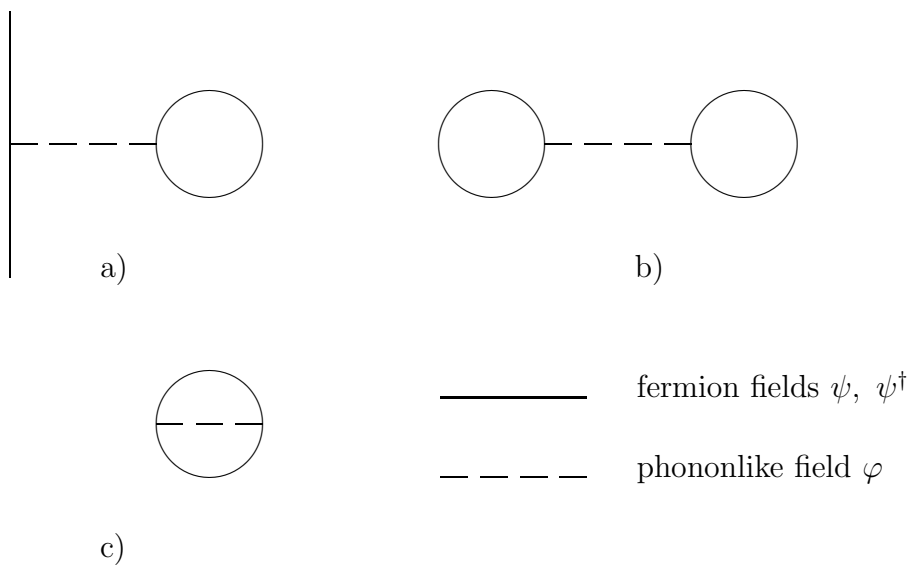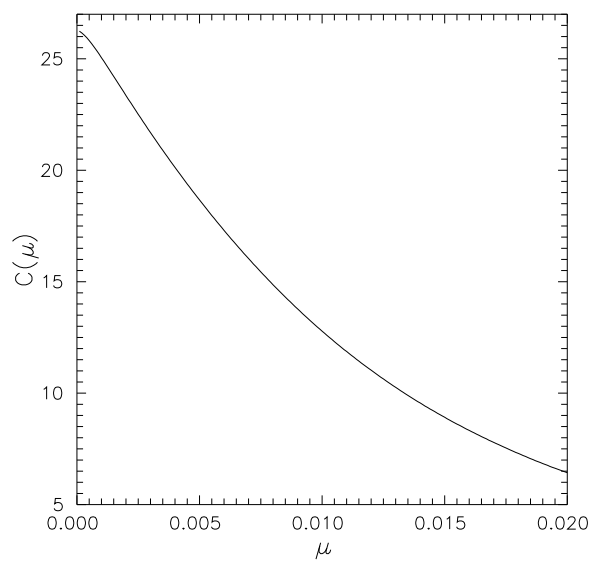
— — — — phononlike field φ

Figure 1: The tadpole graphs.

Figure 2: The function $c(\mu)$ at $N_f = 1$.